Hadronic light-by-light contribution to $(g-2)_\mu$ from lattice QCD

Christoph Lehner (BNL)

RBC and UKQCD Collaborations

October 8, 2015 – Brookhaven Forum 2015

Summer of 2013 – BNL E821 ring to FNAL

SM prediction and experimental status of a_μ

Contribution	Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
QED	11 658 471.895	0.008
EW	15.4	0.1
HVP (Leading-order)	*692.3	4.2
HVP (Higher-order)	-9.84	0.06
Hadronic light-by-light	**10.5	**2.6**
Total SM prediction	11 659 180.3	4.9
BNL E821 result	11 659 209.1	6.3
Fermilab E989 target		$\approx$ **1.6**

* $e^+e^- \rightarrow$ hadrons (exp) and dispersion integrals; "3.3σ tension" based on: K. Hagiwara et al., J. Phys. G38 (2011) 085003: $a_\mu^{\mathrm{HAD,\ LO\ VP}} \times 10^{10} \rightarrow 694.91$

** based on Prades, de Raphael, and Vainshtein 2009 "Glasgow White Paper": QCD model including PS meson contribution; Pauk and Vanderhaeghen Eur.Phys.J. C74 (2014) 8, 3008: include AV,S,T meson poles yields $< 1.0 \times 10^{-10}$ shifts in $a_\mu^{\mathrm{HAD,\ LBL}}$

RBC and UKQCD collaboration on the hadronic light-by-light contribution

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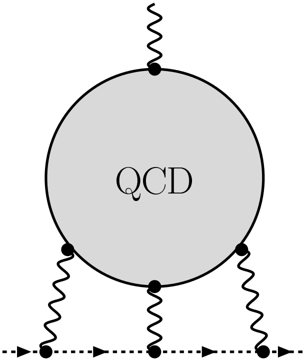
Antonin Portelli (Edinburgh)

Norikazu Yamada (KEK)

For more details, see recent talks at Lattice 2015 by M. Hayakawa, L. Jin, and C.L.

The hadronic light-by-light contribution (HLbL)

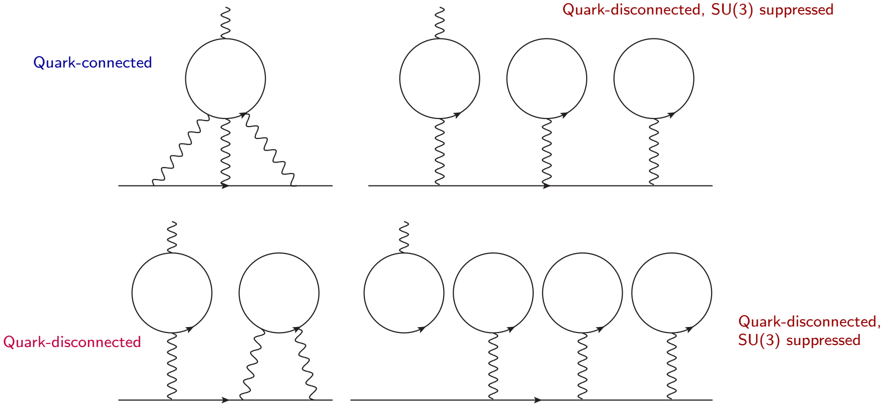


For external photon index μ with momentum q:

$$(-ie)\left[\gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q^\nu}{2m}F_2(q^2)\right] \tag{1}$$

with $F_2(0) = a_\mu$.

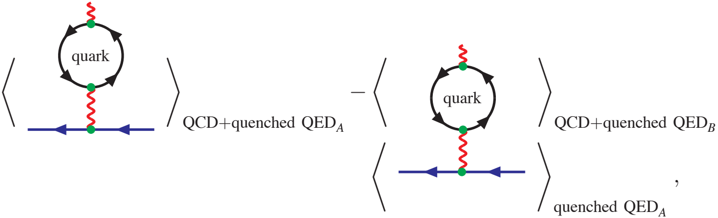
Important lattice terminology – quark-connected diagrams



Representative diagrams with one to four quark loops; gluons not drawn

HLbL – A long-standing problem of interest for our collaboration

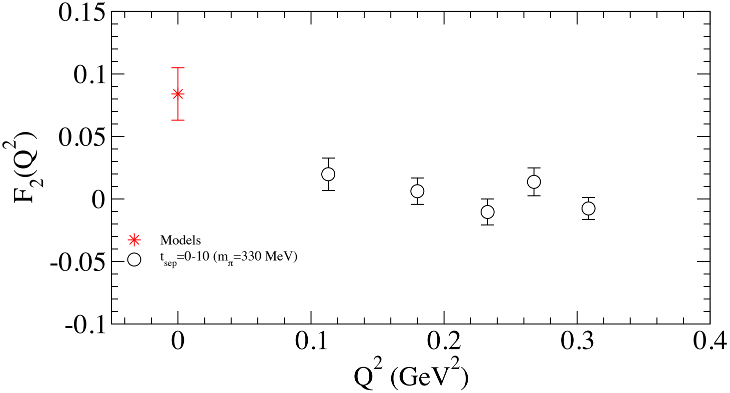
First methodology paper 10 years ago: Blum, Hayakawa, Izubuchi, Yamada: PoS(LAT2005)353; Quark-connected contribution only



Noise control: impose quantum-average properties config-by-config ($e \to -e$, $p \to -p$)

First implementation of this methodology 10 years later:

Blum et al., Phys.Rev.Lett. 114 (2015) 1, 012001: connected diagrams only, $m_\pi = 329$ MeV, $a^{-1} = 1.73$ GeV, $L = 24^3 \times 64$



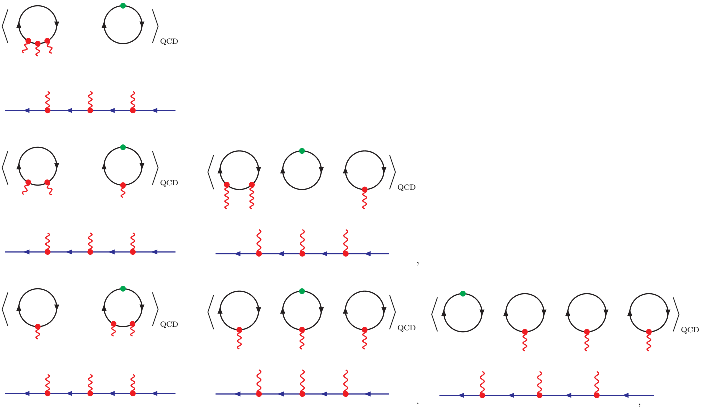
y axis in units of $(\alpha/\pi)^3$

Imperfections that need to be addressed:

- Omission of quark-disconnected diagrams
- Control of large QED finite-volume errors
- Direct evaluation of / extrapolation to F_2 at $Q^2 = 0$
- Control of excited state contributions
- Computation at physical pion mass

Inclusion of QCD+dynamical QED

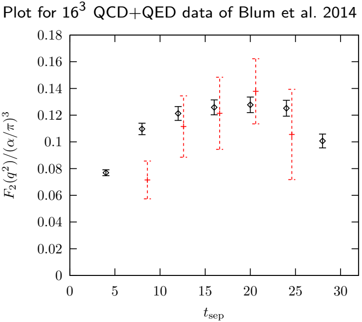
Blum, Hayakawa, and Izubuchi, PoS(LATTICE 2013)439; Update: M. Hayakawa Lattice 2015



Addresses disconnected diagrams, however, isolation of signal from noise is challenging

Re-examine statistics

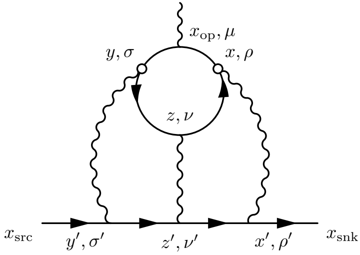
QCD+QED simulations suffer from large statistical uncertainties. We explore a different method here:

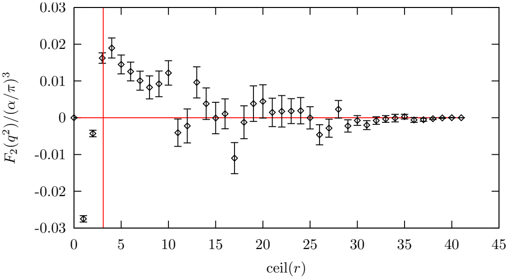


Luchang Jin

Same-cost comparison: red data: old method QCD+quenched QED, black: new stochastic sampling method (Luchang Jin talk at lattice 2015)

New stochastic sampling method



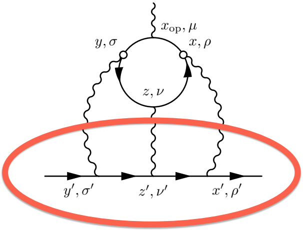


Stochastically evaluate the sum over vertices x and y:

- Pick random point x on lattice
- Sample all points y up to a specific distance $r = |x - y|$, see vertical red line
- Pick y following a distribution $P(|x - y|)$ that is peaked at short distances

Advantage: order of magnitude smaller noise, Disadvantage: disconnected diagrams by hand

QCD + QED on a lattice – finite-volume errors



Need to sum over all displacements between QCD and QED part to control FV errors.

Since muon line does not couple to gluons, this can be done in a straightforward way: C.L. talk at lattice 2015

Direct evaluation of form-factor at $F_2(Q^2 = 0)$

Model of problem: The lattice gives the position-space correlator $C(x)$ whose momentum space version

$$C(q) = \sum_x e^{iqx} C(x) \tag{2}$$

vanishes for $q = 0$ and the observable is related to

$$F = \lim_{q \to 0} \frac{C(q)}{q}\,, \tag{3}$$

while the lattice only has access to $C(q)$ for finite-volume quantized momenta q.

However: if $C(x) \to 0$ sufficiently fast as $|x| \to \infty$, we can write

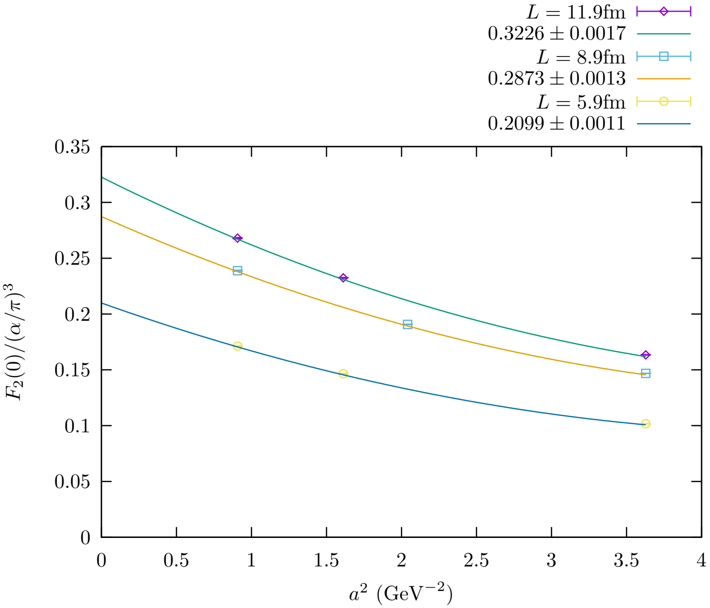
$$F = i \sum_x x\, C(x) \tag{4}$$

with controlled finite-volume errors.

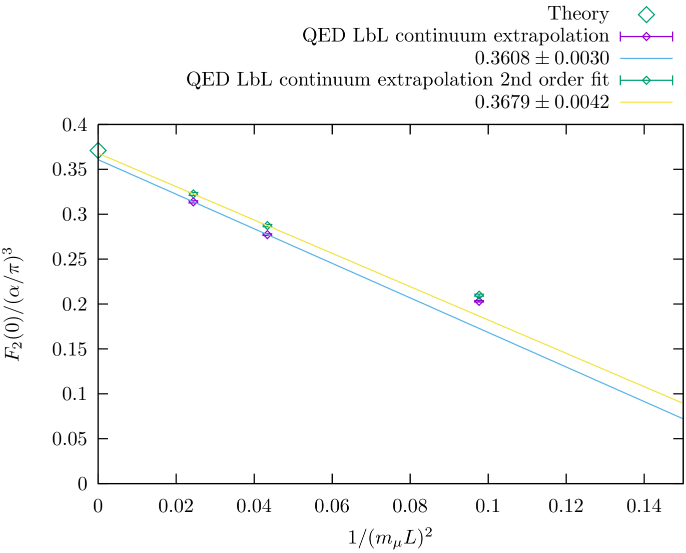
Imperfections that need to be addressed:

- Omission of quark-disconnected diagrams
- ✓ Control of large QED finite-volume errors
- ✓ Direct evaluation of / extrapolation to F_2 at $Q^2 = 0$
- ✓ Control of excited state contributions
- ✓ Computation at physical pion mass

Demonstration of validity – Replace quark with lepton loop



Demonstration of validity – Replace quark with lepton loop



Lattice result nicely extrapolates to the known analytic theory result;
Note that the difference between the lepton and full computation is merely the quark-propagator used, this is a strong test!

Status of lattice hadronic light-by-light determination:

- Quark-connected diagram seems to be controllable with current methodology
- We are currently running a large-scale computation at Argonne National Laboratory using 175M core hours (≈ 5000 typical laptop years) with precision-target for the quark-connected diagram of $10\% - 20\%$. Preliminary results hint at a statistical error of below 10% at the end of the run.

Work in progress:

- Quark-disconnected diagram strategy is being optimized. This is a statistics problem not a systematic one! Starting with diagram surviving in SU(3) limit.

Other collaborations have started similar efforts (Mainz group presented a computation of the quark four-point function at lattice 2015).

The lattice community is actively putting its focus on this important quantity.

Thank you

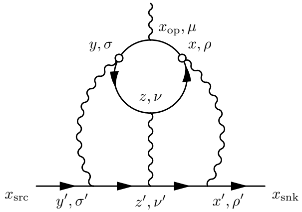
Backup slides

Excited states – A quick reminder of further lattice methodology

The lattice can compute Euclidean-space correlation functions. We extract operator matrix elements by taking large time separations to isolate on-shell contributions. Example:

$$\begin{aligned}\langle A(t)O(t_{\rm op})B(0)\rangle &= \sum_{n,m}\langle A|n\rangle\langle n|O|m\rangle\langle m|B\rangle e^{-E_n(t-t_{\rm op})}e^{-E_m t_{\rm op}}\\ &\to \langle A|n_0\rangle\langle n_0|O|m_0\rangle\langle m_0|B\rangle e^{-E_{n_0}(t-t_{\rm op})}e^{-E_{m_0}t_{\rm op}}\,.\end{aligned}$$

Replacing $O(t_{\rm op}) \to e^{iqt_{\rm op}}$ allows for determination of norm and to extract $\langle n_0|O|m_0\rangle$.



Excited states

- As we go to larger volumes, excited state contributions of $\mu + \gamma$ etc. may be enhanced

- Lattice QED perturbation theory converges well and can be used to construct improved source

- We are exploring this with the *PhySyHCAl* system that also was used for a free-field test of Blum et al. 2014